99. A highway curve of radius $R$ is banked at an angle $\theta$. The coefficient of static friction between the road surface and the tires of a sports car is $\mu_s$.

a) At what constant speed should the sports car round the curve for minimum tire wear?

b) Show that the minimum speed at which the curve can be safely negotiated without the car "skidding sideways" is:

$$v_{\text{min}} = \sqrt{Rg \frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta}}$$

a) For "design" speed: $f_s = 0$

$$N \sin \theta = \frac{mv_0^2}{R} \quad (1)$$

$$N \cos \theta = mg \quad (2)$$

Dividing (1) by (2),

$$\frac{v_0^2}{Rg} = \tan \theta$$

$$v_0 = \sqrt{Rg \tan \theta}$$

b) At $v < v_0$ the inward component of the normal force is more than required to provide the required centripetal force and must be reduced by the sideways friction $f_s$, directed up the bank. In that case:

$$N \sin \theta - f_s \cos \theta = \frac{mv^2}{R} \quad (3)$$

$$N \cos \theta + f_s \sin \theta - mg = 0 \quad (4)$$

Rearrange and divide:

$$\frac{v^2}{Rg} = \frac{N \sin \theta - f_s \cos \theta}{N \cos \theta + f_s \sin \theta}$$

when $v = v_{\text{min}}$, $f_s = f_{s(\text{max})} = \mu_s N$

$$\therefore \quad v_{\text{min}} = \sqrt{Rg \frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta}}$$
111. Three masses $m_1 = 2.0 \text{ kg}$, $m_2 = 6.0 \text{ kg}$ and $m_3 = 3.0 \text{ kg}$ are arranged as in the figure. Mass $m_1$ is held in place by a horizontal string and masses $m_2$ and $m_3$ are connected by a string over a massless, frictionless pulley. After being released, mass $m_3$ is observed to descend. If the coefficient of kinetic friction at all sliding surfaces is $\mu_k = 0.15$, determine the accelerations of $m_2$ and $m_3$.

Free Body Diagram

\[
a_1 = 0
\]
\[
m_ia_i = 0 = f_1 - T_1
\]
\[
N_1 = m_1g, \quad f_1 = \mu_k N_1 = \mu_k m_1 g
\]
\[
m_2a_2 = T_2 - f_1 - f_2
\]
\[
N_2 = N_1 + m_3g = (m_1 + m_2)g, \quad f_2 = \mu_k N_2 = \mu_k (m_1 + m_2)g
\]
\[
\therefore \quad m_2a_2 = T_2 - \mu_k (2m_1 + m_2)g
\]
\[
m_3a_3 = m_3g - T_2, \quad \text{i.e.} \quad T_2 = m_3g - m_3a_3
\]
Using $a_1 = a_3 = a$, we have
\[
m_2a = m_3g - m_3a - \mu_k (2m_1 + m_2)g
\]
\[
a = \frac{m_3g - \mu_k (2m_1 + m_2)g}{m_2 + m_3} = 1.63 \text{ m/s}^2.
\]
62. (a) The tangential acceleration is the time derivative of the speed.

\[ a_{\text{tan}} = \frac{dv_{\text{tan}}}{dt} = \frac{d\left(3.6 + 1.5t^2\right)}{dt} = 3.0 \quad \rightarrow \quad a_{\text{tan}}(3.0 \text{s}) = 3.0(3.0) = 9.0 \text{ m/s}^2 \]

(b) The radial acceleration is given by Eq. 5-1.

\[ a_{\text{rad}} = \frac{v_{\text{tan}}^2}{r} = \frac{\left(3.6 + 1.5t^2\right)^2}{r} \quad \rightarrow \quad a_{\text{rad}}(3.0 \text{s}) = \frac{\left(3.6 + 1.5(3.0)^2\right)^2}{22 \text{ m}} = 13 \text{ m/s}^2 \]

63. We show a top view of the particle in circular motion, traveling clockwise. Because the particle is in circular motion, there must be a radially-inward component of the acceleration.

(a) \( a_r = a \sin \theta = \frac{v^2}{r} \quad \rightarrow \quad v = \sqrt{ar \sin \theta} = \sqrt{(1.15 \text{ m/s}^2)(3.80 \text{ m}) \sin 38.0^\circ} = 1.64 \text{ m/s} \)

(b) The particle’s speed change comes from the tangential acceleration, which is given by \( a_{\text{tan}} = a \cos \theta \). If the tangential acceleration is constant, then using Eq. 2-12a,

\[ v_{\text{tan}} - v_{0 \text{tan}} = a_{\text{tan}}t \quad \rightarrow \quad v_{\text{tan}} = v_{0 \text{tan}} + a_{\text{tan}}t = 1.64 \text{ m/s} + \left(1.15 \text{ m/s}^2\right)(\cos 38.0^\circ)(2.00 \text{ s}) = 3.45 \text{ m/s} \]